CBCS SCHEME

USN | | | | | | | | | | **18EC54**

Fifth Semester B.E. Degree Examination, Feb./Mar.2022
Information Theory and Coding

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define self information. Why logarithmic expression is chosen for measuring information. **(04 Marks)**

b. (i) Find relationship between Hartleys, nats and bits.

(ii) A discrete source emits one of the four symbols S_0, S_1, S_2 and S_3 with probabilities $1/3$, 1/6, 1/4 and 1/4 respectively. The successive symbols emitted by the source are statistically independent. Calculate the entropy of the source. **(08 Marks)**

c. (i) State the properties of entropy.

(ii) A source transmits two independent messages with probabilities of P and $1-P$ respectively. Prove that the entropy is maximum when both the message are equally likely. Plot the variation of entropy (H) as a function of probability 'P' of the messages. **(08 Marks)**

OR

2 a. Consider the following Markov source shown in Fig. Q2 (a). Find the

(i) State probabilities (ii) State entropies.
(iii) Source entropy. (iv) G_1, G_2
(v) Show that $G_1 > G_2 > H$ **(10 Marks)**

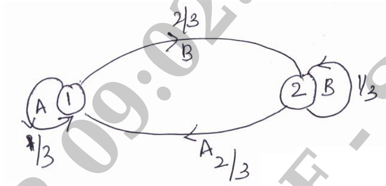

Fig. Q2 (a)

b. Consider a zero memory source emitting three symbols x, y and z with respective probabilities $\{0.6, 0.3, 0.1\}$. Calculate

(i) Entropy of the source.
(ii) All symbols and the corresponding probabilities of the second order extension of the source. Find the entropy of the second-order extension of the source.
(iii) Show that $H(s^2) = 2 * H(s)$ **(10 Marks)**

Module-2

3 a. The table 3.1 below provides codes for five different symbols. Identify which of the following codes are prefix codes. Also draw the decision diagram for the prefix codes. **(04 Marks)**

Code A	Code B	Code C	Code D
0	1	00	10
10	01	110	111
110	111	1110	110
1110	10	001	01
111	00	011	00

b. Apply Shannon's encoding algorithm to the following set of messages and obtain code efficiency and redundancy. **(10 Marks)**

m_1	m_2	m_3	m_4	m_5
$\frac{1}{8}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{8}$

c. Construct a Binary code by applying Huffman encoding procedure for the following messages with respective probabilities of 0.4, 0.2, 0.2, 0.1, 0.07 and 0.03. Also determine the code efficiency and redundancy of the code. **(06 Marks)**

OR

4 a. Design a Trinary source code for the source shown using Huffman's coding procedure:

$$S = \{S_1\ S_2\ S_3\ S_4\ S_5\ S_6\}$$

$$P = \left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{12}, \frac{1}{12}\right\}.$$

(10 Marks)

b. Consider a source $S = \{S_1, S_2\}$ with probabilities $3/4$ and $1/4$ respectively. Obtain Shannon-Fano code for source S, its 2nd and 3rd extension. Calculate efficiencies for each case and justify the results. **(10 Marks)**

Module-3

5 a. What is mutual information? Mention its properties. **(06 Marks)**

b. A transmitter has an alphabet consisting of 5 letters $\{a_1, a_2, a_3, a_4, a_5\}$ and the receiver has an alphabet of four letters $\{b_1, b_2, b_3, b_4\}$. The joint probabilities of the system are given below:

$$P(A, B) = \begin{array}{c c} & \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} & \begin{pmatrix} 0.25 & 0 & 0 & 0 \\ 0.10 & 0.30 & 0 & 0 \\ 0 & 0.05 & 0.10 & 0 \\ 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{pmatrix} \end{array}$$

Compute different entropies of the channel. **(08 Marks)**

c. For the channel matrix shown, find the channel capacity.

$$P\left(\frac{b_j}{a_i}\right) = \begin{array}{c c} & \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \end{pmatrix} \end{array}$$

(06 Marks)

OR

6 a. In a communication system a transmitter has 3 input symbols $A = \{a_1, a_2, a_3\}$ and receiver also has 3 output symbols $B = \{b_1, b_2, b_3\}$. The matrix given below shows joint probability matrix with some marginal probabilities. **(06 Marks)**

a_i \ b_j	b_1	b_2	b_3
a_1	$\frac{1}{12}$	*	$\frac{5}{36}$
a_2	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{5}{36}$
a_3	*	$\frac{1}{6}$	*
$P(b_j)$	$\frac{1}{3}$	$\frac{14}{36}$	*

(i) Find the missing probabilities (*) in the table.

(ii) Find $P(b_3/a_1)$ and $P(a_1/b_3)$.

(iii) Are the events a_1 and b_1 statistically independent? Why?

b. Find the capacity of the channel shown in the Fig. Q6 (b) below using Murugo's method. **(08 Marks)**

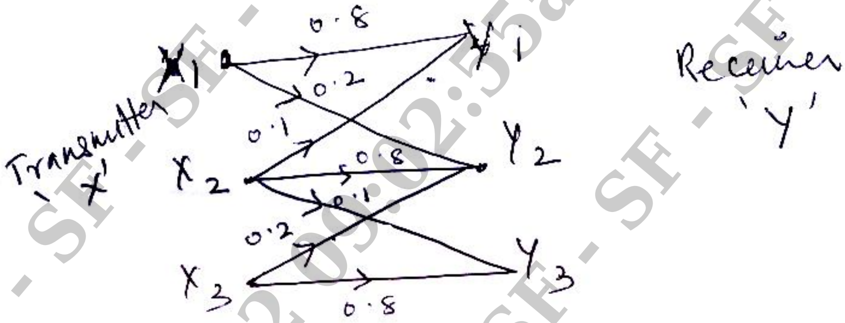

Fig. Q6 (b)

c. Discuss Binary Ensure channel and derive channel capacity equation. **(06 Marks)**

Module-4

7 a. For a systematic (7, 4) linear block code, the parity matrix P is given by,

$$[P] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(i) Find all possible code vectors.

(ii) Draw the encoding circuit.

(iii) Draw the syndrome calculation circuit. **(10 Marks)**

b. Design an encoder for a (7, 4) binary cyclic code generated by $g(x) = 1 + x + x^3$ and verify its operation using the message vectors (1 0 0 1) and (1 0 1 1). **(10 Marks)**

OR

8 a. Define G and H matrix and show that $GH^T = 0$. **(05 Marks)**

b. The Parity check bits of a (8, 4) block code are generated by,

$C_5 = d_1 + d_2 + d_4$
$C_6 = d_1 + d_2 + d_3$
$C_7 = d_1 + d_3 + d_4$
$C_8 = d_2 + d_3 + d_4$

where d_1, d_2, d_3, d_4 are the message bits.

(i) Find the generator and parity matrix for this code.
(ii) Find the minimum weight.
(iii) Show that its capable of correcting all single error pattern and capable of detecting double errors by preparing the syndrome table for them. **(10 Marks)**

c. Design a linear block code with minimum distance $d_{min} = 3$ and message length of 4 bits. **(05 Marks)**

Module-5

9 a. With a neat block diagram, draw a general decoding circuit for a linear block code. Also draw the complete error correcting circuit for a (7, 4) linear block code if the error bits are given in terms of the syndrome bits as given in equation below:

$S = [S_1\ S_2\ S_3] = [(r_1 + r_2 + r_3 + r_5), (r_1 + r_2 + r_4 + r_6), (r_1 + r_3 + r_4 + r_7)]$. **(06 Marks)**

b. Consider a (7, 4) cyclic code with $g(x) = 1 + x + x^3$. Obtain the code polynomial in non systematic and systematic form for the input sequence.

(i) 1 0 1 0
(ii) 1 1 0 0 **(10 Marks)**

c. Write short notes on BCH codes. **(04 Marks)**

OR

10 a. For a (2, 1, 3) convolutional encoder with $g^{(1)} = 1\ 0\ 1\ 1$ and $g^{(2)} = 1\ 1\ 1\ 1$. Find the output sequence using the two following approaches:

(i) Time domain approach.
(ii) Transform domain approach.

Also draw the encoder diagram. **(10 Marks)**

b. For a (2, 1, 2) convolutional encoder with $g^{(1)} = [1\ 1\ 1]$, $g^{(2)} = [1\ 0\ 1]$

(i) Draw the transition table.
(ii) State diagram.
(iii) Draw code tree.
(iv) Using the code tree, find the encoded sequence for the message 1 0 1 1 1.
(v) Draw the Trellis diagram. **(10 Marks)**

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